

# **Giant Resonances**

## **First Experiments:**

G.C.Baldwin and G.S. Klaiber, Phys.Rev. 71, 3 (1947)

General Electric Research Laboratory, Schenectady, NY

## **Theoretical Explanation:**

M. Goldhaber and E. Teller, Phys. Rev. 74, 1046(1948)

University of Illinois      University of Chicago

**It is an oscillation of the neutrons against the protons!**

Think electric dipole

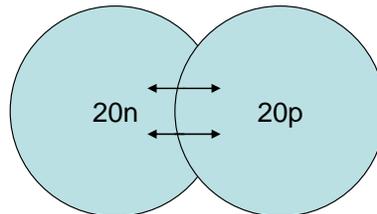
Positive “charge” is the collection of protons.

Negative “charge” is the collection of neutrons.

**The Isovector Giant Dipole Resonance**

(this E. Teller is Edward Teller – think Hydrogen bomb)

$^{40}\text{Ca}$



Isovector Giant Dipole Resonance

**Analogous to charge distributions**

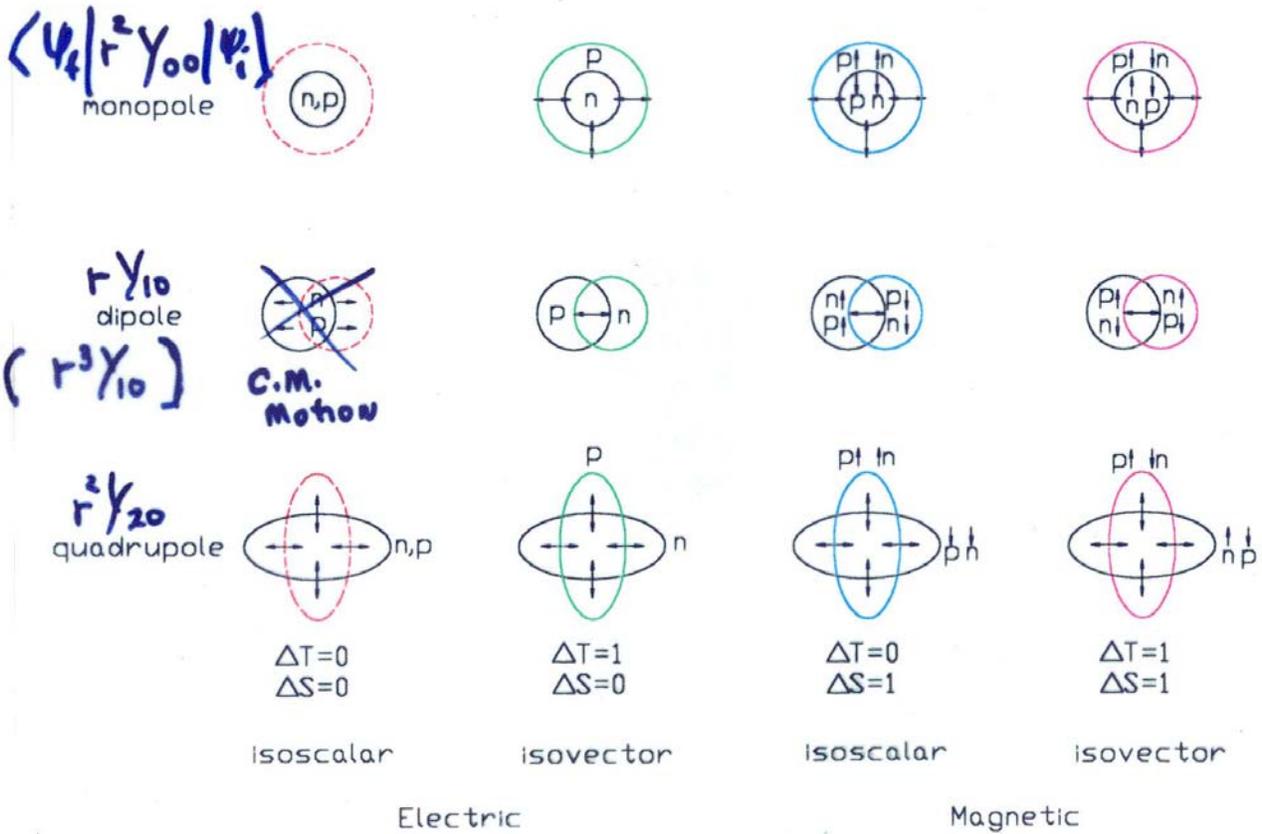
<b>Monopole</b>	<b>L=0</b>
<b>Dipole</b>	<b>L=1</b>
<b>Quadrupole</b>	<b>L=2</b>
<b>Octupole</b>	<b>L=3</b>
<b>Etc.</b>	

**There should be lots of “Giant Resonances”**

**Isovector** (n's and p's  $180^\circ$  out of phase)  
**Isoscalar** (n's and p's in phase)  
**Spin** (spin up and down  $180^\circ$  out of phase)

# Shape Oscillations

Qualitative picture of giant resonance modes of the nucleus



## “Giant Resonance”

1. Many Nucleons involved in motion ( $\sim 10$  in  $^{40}\text{Ca}$ ).
2. Contains “most of” strength for that multipole.

## Limit on strength for each multipole.

Electromagnetic Operator  $Q_L = \sum_i r_i^L Y_{LM}(\theta_i)$

$Y_{LM}$  is a spherical harmonic  
for multipole L, magnetic substate M.

$\sum_n E_n |\langle n | Q_L | 0 \rangle|^2 = S_L$  Energy Weighted Sum Rule

if there are no velocity dependent forces.

$E_n$  is excitation energy of state  
 $|0\rangle$  represents wave function of ground state  
 $\langle n|$  represents wave function of excited state

For  $L \geq 2$   $S_L = (\hbar^2 L(2L+1)A)/2\pi m * \langle r^{2L-2} \rangle$

“Giant Resonance”

Contains ~90-99% of this strength

Rest of strength in low lying states of nucleus.

**1970: Only Dipole had been observed.**

**Theorists: Came up with reasons why others weren't there!**

**1971:**

**Graduate Student: Ranier Pitthan**

**Advisor: Thomas Walcher**

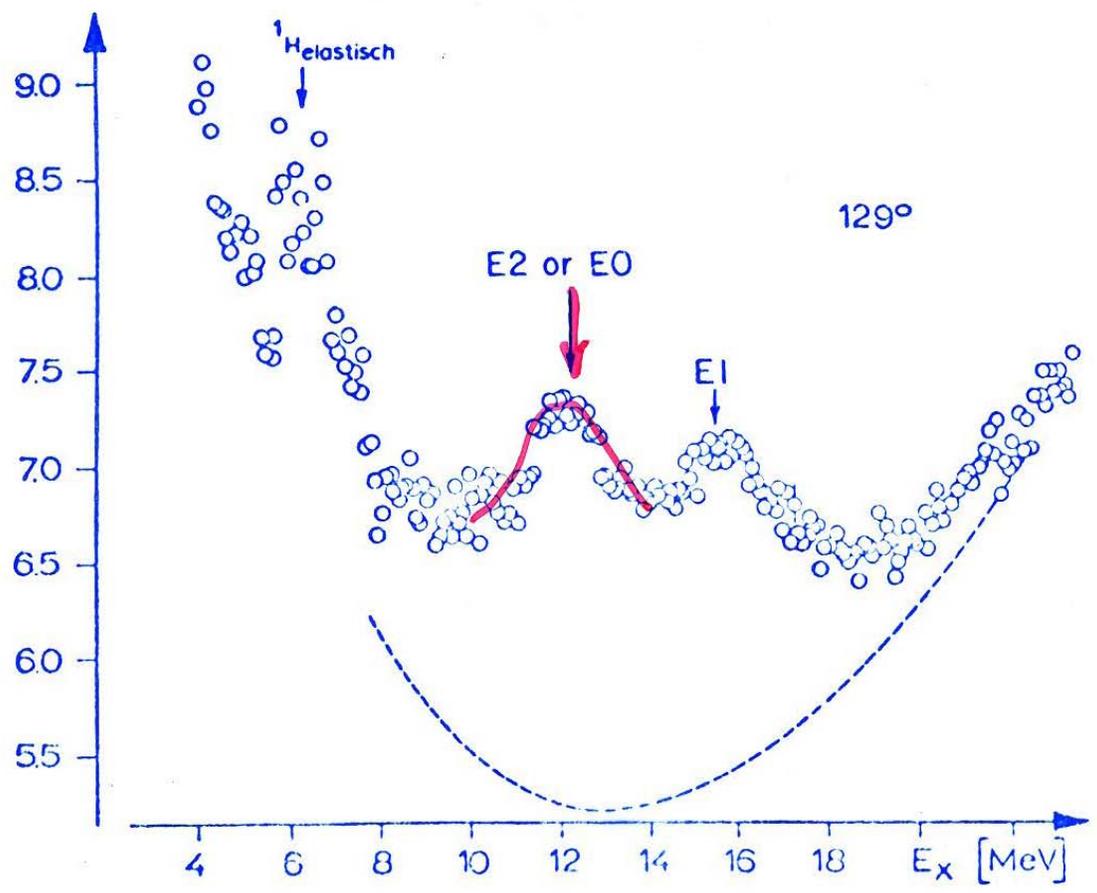
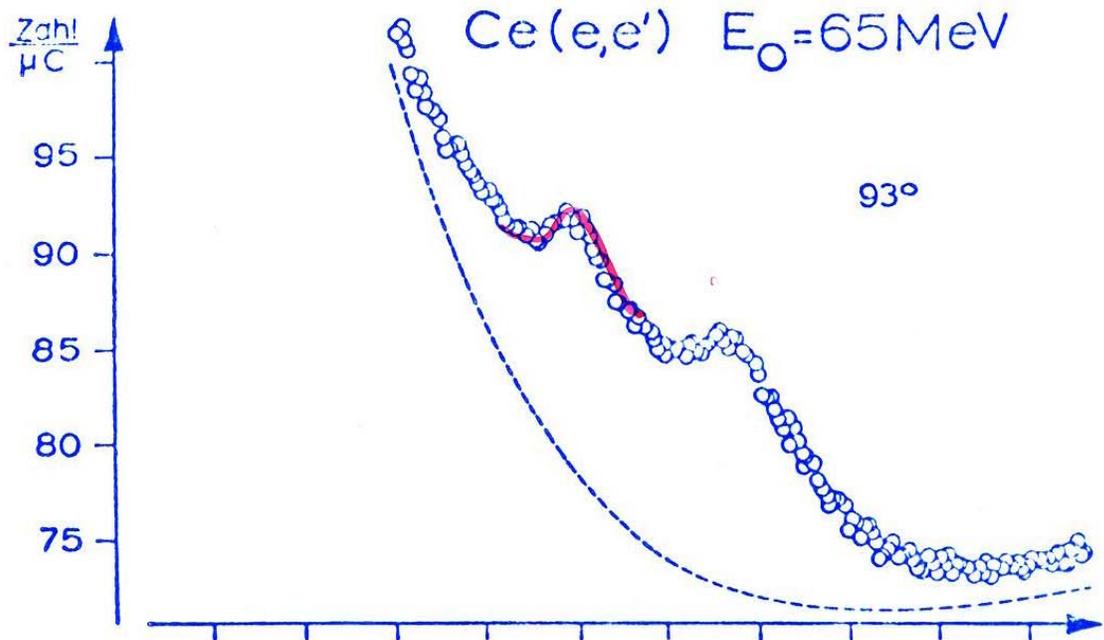
**Electron scattering off of Ce, La, Pr targets**

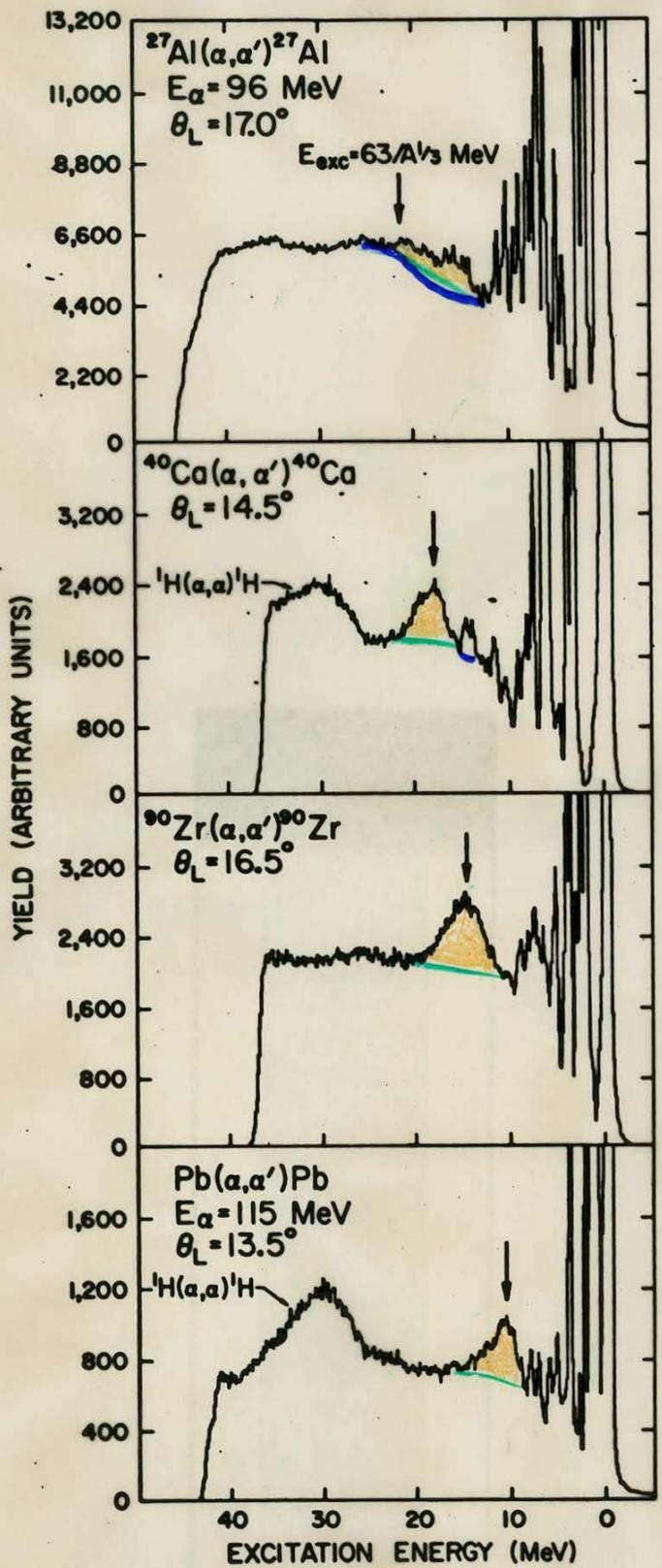
**Darmstadt, Germany**

**Electrons interact only with the protons  
(no nuclear force)**

**Excite Isoscalar and Isovector states ~ same.**

R. Pitthan and T. Walcher, Phys. Lett. 36B 563 (1971)

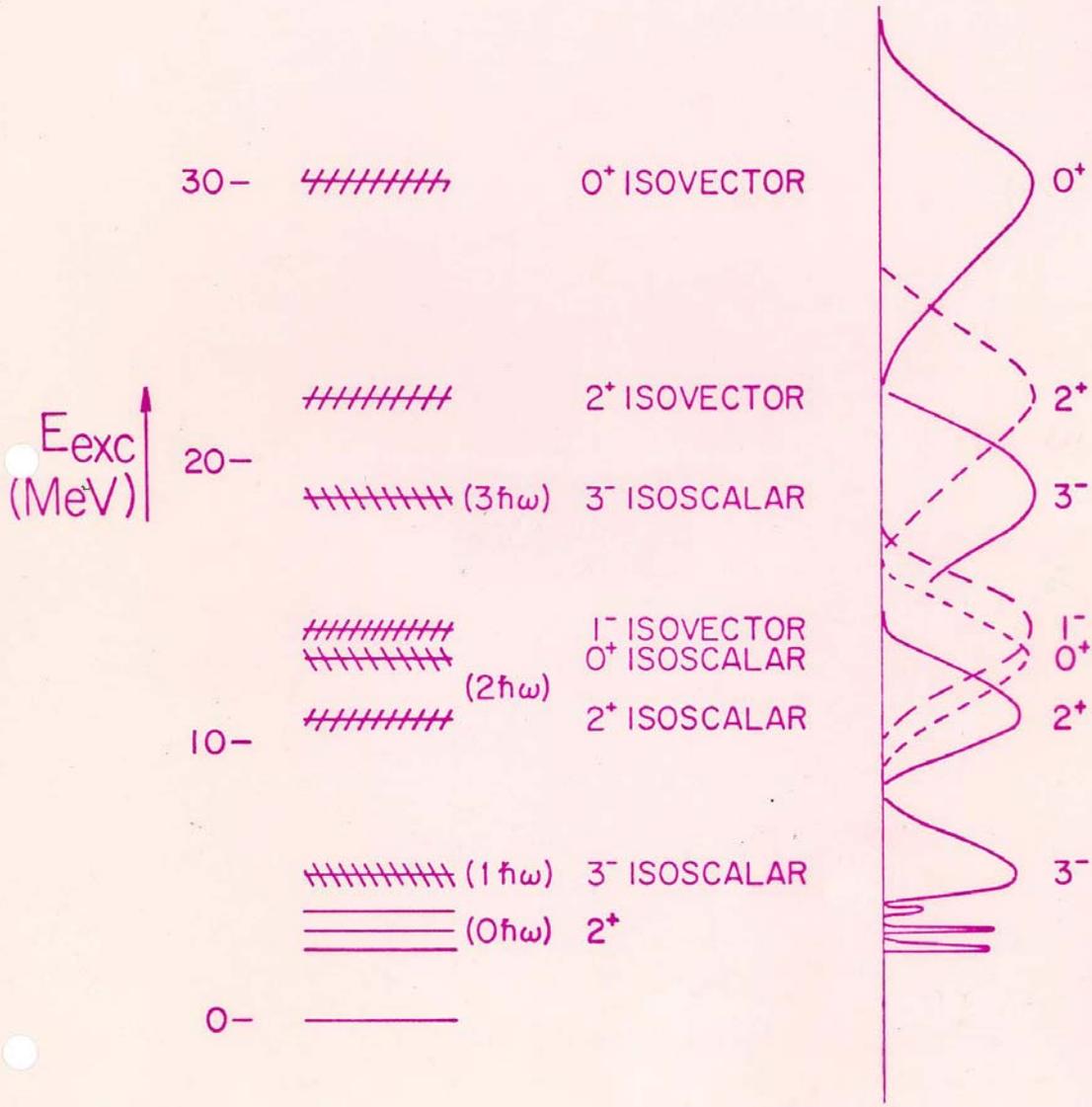




Texas A&M

1973

# STRENGTH DISTRIBUTION A ~ 200



**One in particular:**

**Isoscalar Giant Monopole Resonance**

**The “breathing mode”**

**A 3 dimensional harmonic oscillator**

SHO:  $\omega = (k/m)^{1/2}$  and  $E = \hbar\omega$

**Liquid drop model of nucleus:**

$$E_{\text{GMR}} = (\pi\hbar / 3r_0 A^{1/3}) * (K_A / m)^{1/2}$$

$r_0$  is nuclear radius

$A$  is #(protons + neutrons)

$m$  is mass of nucleon

$K_A$  is compressibility of nucleus

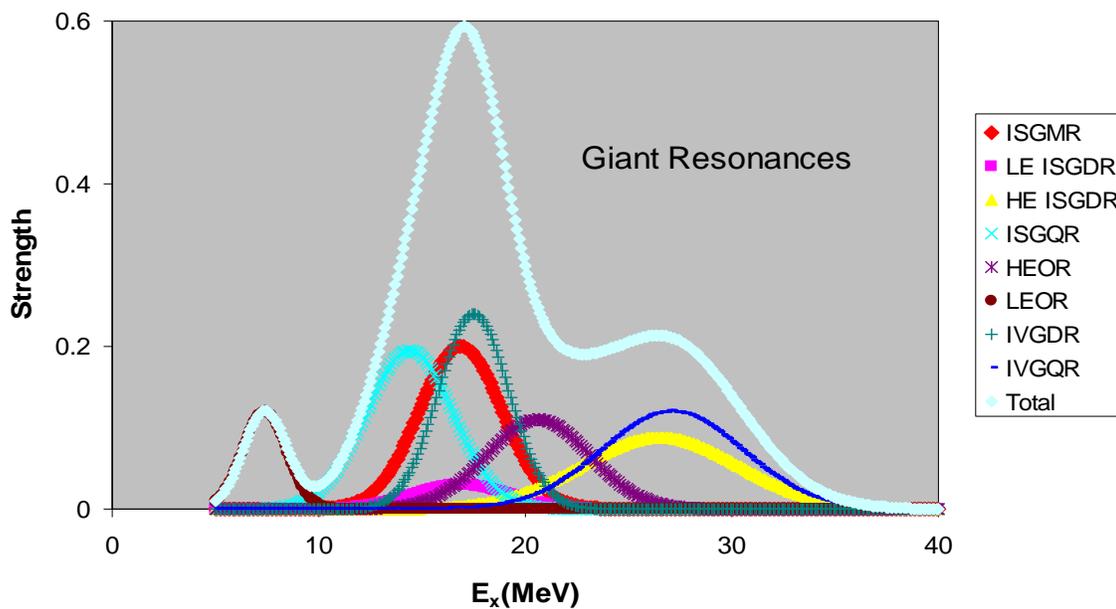
**Can get compressibility of nucleus from ISGMR**

**IF WE CAN FIND IT!**

## Strength of Giant Resonances

Light Blue is what you see (the sum of strengths).

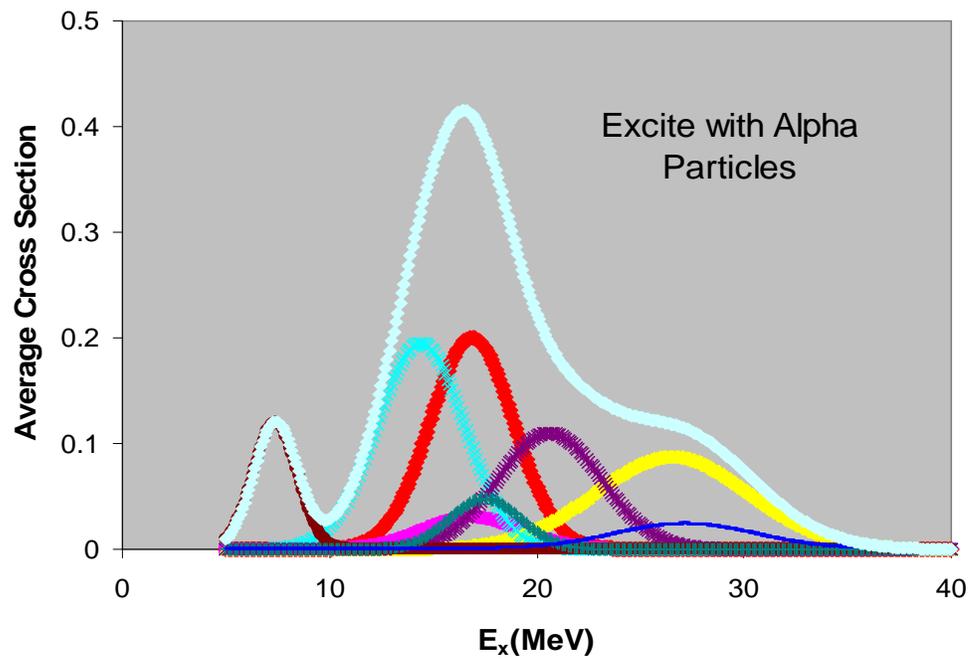
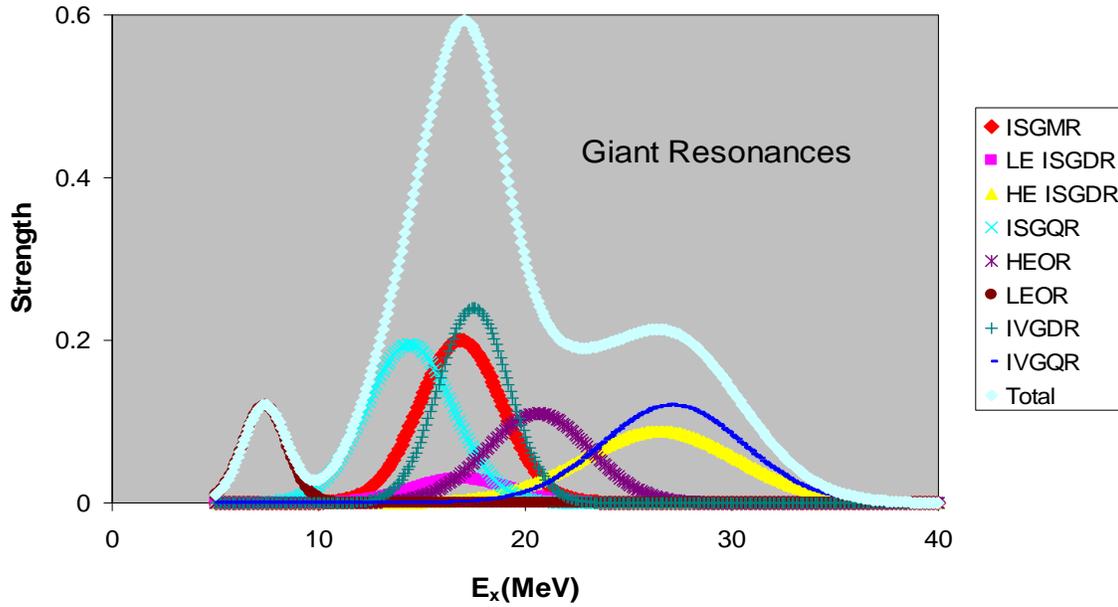
**Red is the Isoscalar Giant Monopole Resonance**  
(what you want to measure)



**How do you separate out the Monopole??**

**Use an alpha particle beam to excite them!**

**Alpha particle (2p, 2n): excites Isoscalar states strongly.  
Isovector states weakly.**



**Need to do more!**

## Diffraction Model of $\alpha$ inelastic Scattering

Due to highly absorptive nature of  $\alpha$ -Nucleus interaction, can treat it as scattering by a black disc. (Fraunhofer scattering)

See Bernstein (Adv. Nucl. Phys. 3, 1969)

This is Blair Model (1959, 1965).

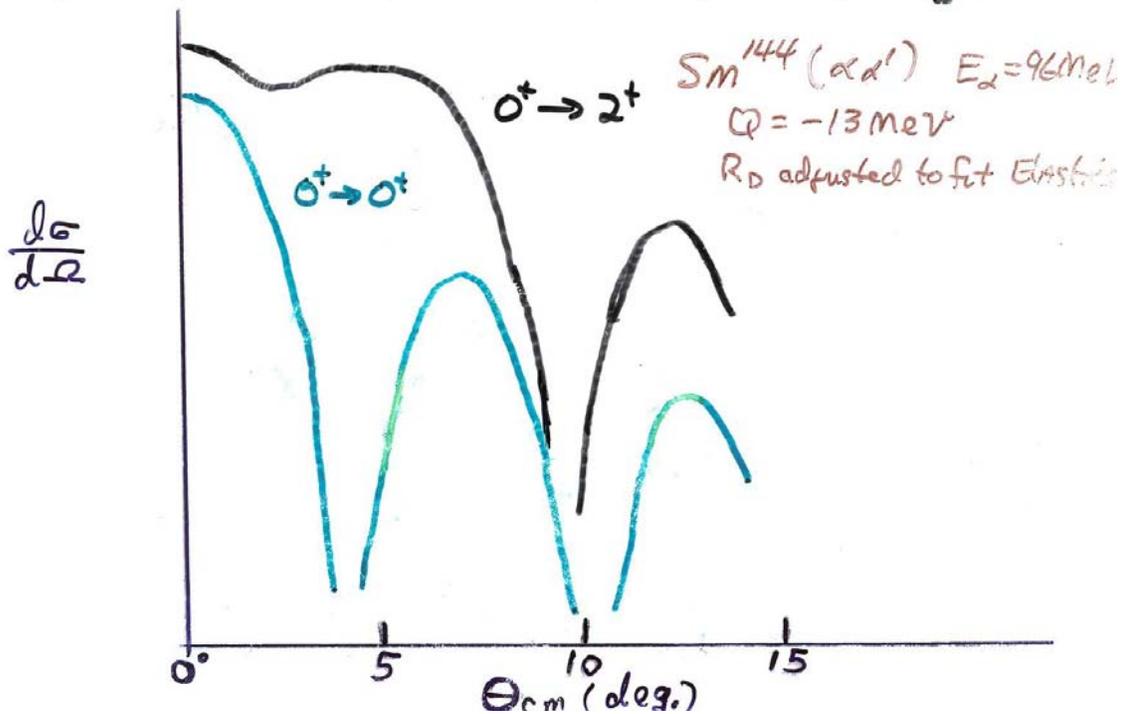
Cross-section given by square of Bessel function:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{elastic}} \propto |J_1(qR_D)|^2$$

$q$  is momentum Transf.  
 $R_D$  is diffraction Radius

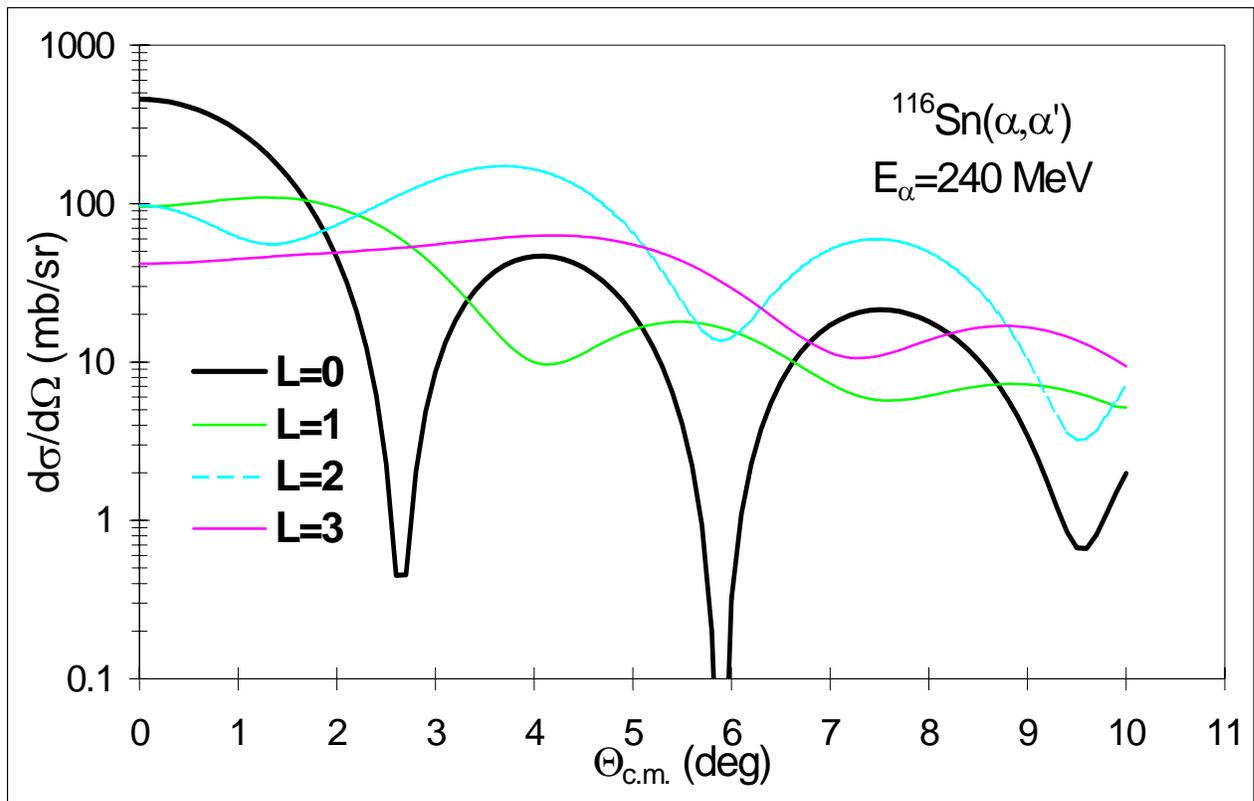
$$\left(\frac{d\sigma}{d\Omega}\right)_{0^+ \rightarrow 0^+} \propto |J_0(qR_D)|^2$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{0^+ \rightarrow 2^+} \propto \left[\frac{1}{4} J_0^2(qR_D) + \frac{3}{4} J_2^2(qR_D)\right]$$



# Distorted Wave Born Approximation Calculation

Inelastic  $\alpha$  scattering  $E_\alpha=240\text{MeV}$

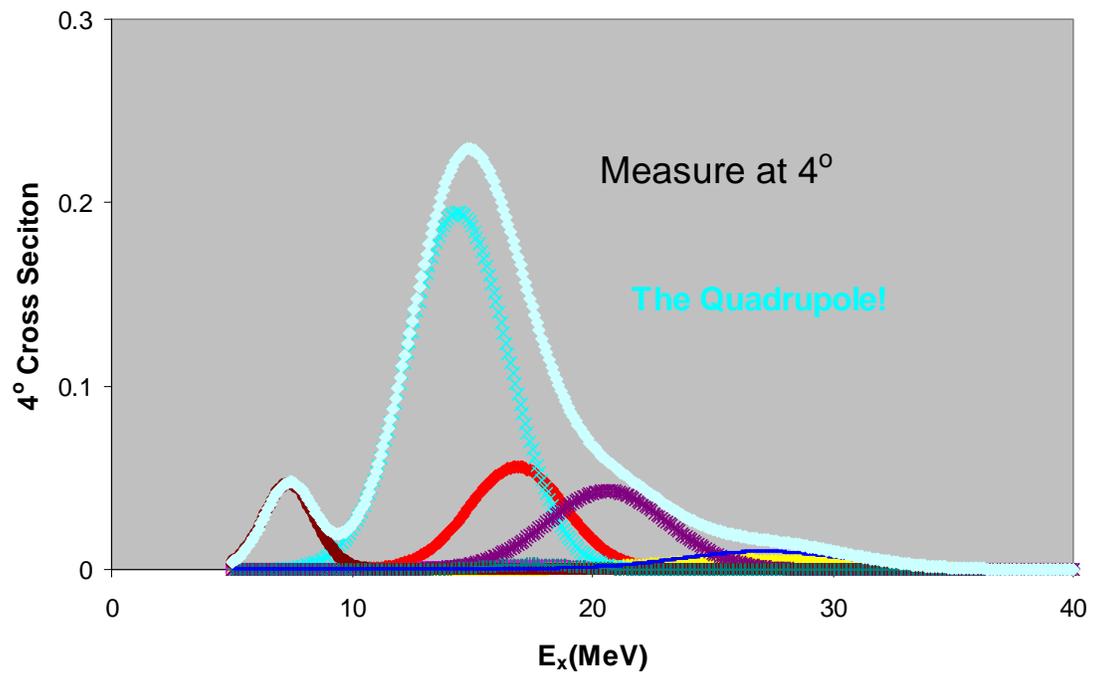
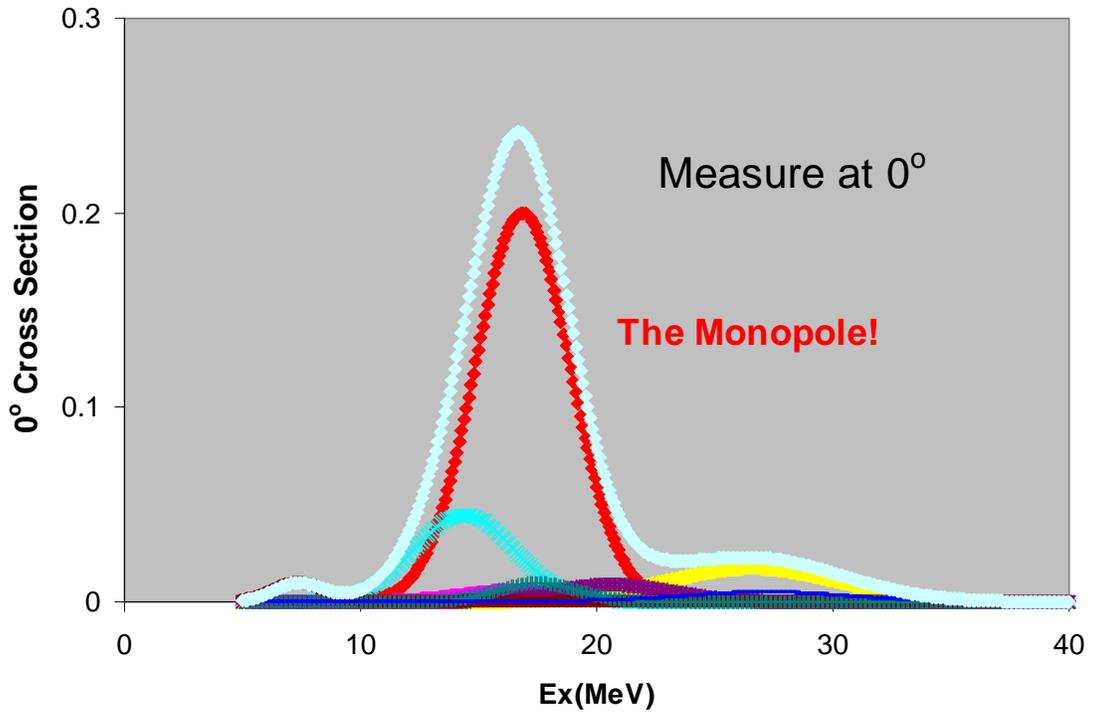


Measure at different scattering angles!

Could separate Monopole from Quadrupole

by measuring  $1.5^\circ$  to  $4^\circ$

**Monopole Enhanced at 0°.**



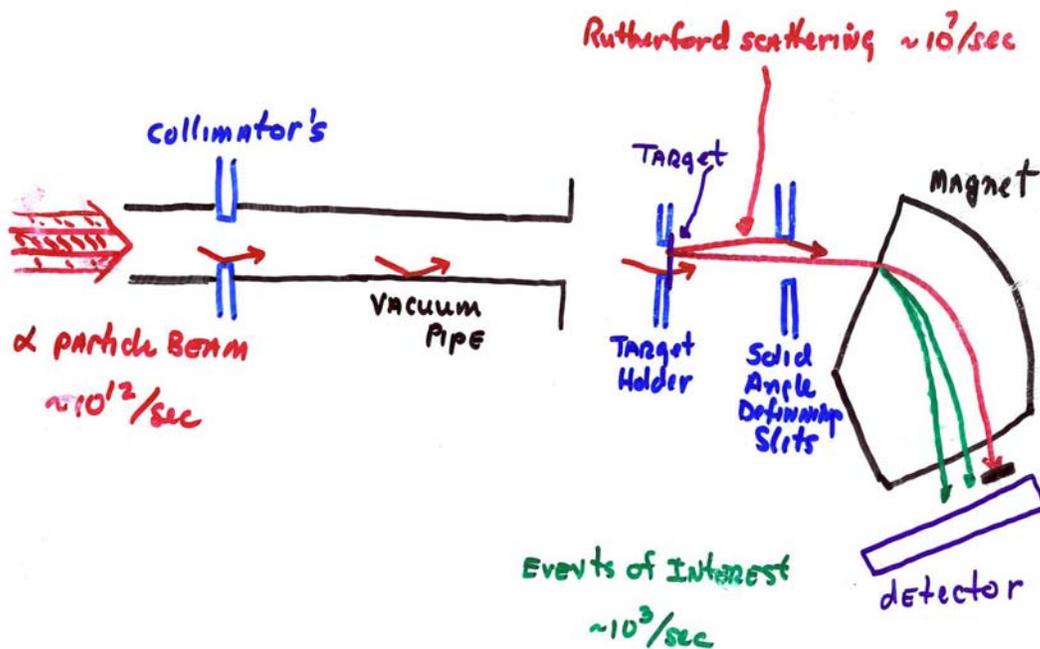
At Small angles

Beam would destroy detectors.

Use Magnet to separate beam from inelastic scattering

BUT!

Problems with small ANGLE ( $\alpha, \alpha'$ )



Beam must be transported without hitting anything!

Nobody had done  $0^\circ$  inelastic scattering.

# Measure over the minimum in the monopole.

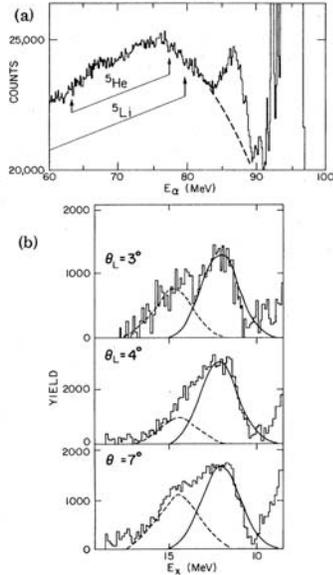


FIG. 1. (a) A  $^{208}\text{Pb}(\alpha, \alpha')$  spectrum taken at  $\theta_L = 4^\circ$ . The dashed line indicates the background chosen. (b) A portion of  $^{144}\text{Sm}(\alpha, \alpha')$  spectra ( $E_\alpha = 96$  MeV) taken at  $3^\circ$ ,  $4^\circ$ , and  $7^\circ$  are shown after subtraction of the continuum background. Gaussian peaks are shown for both components utilizing positions and widths from Table I.

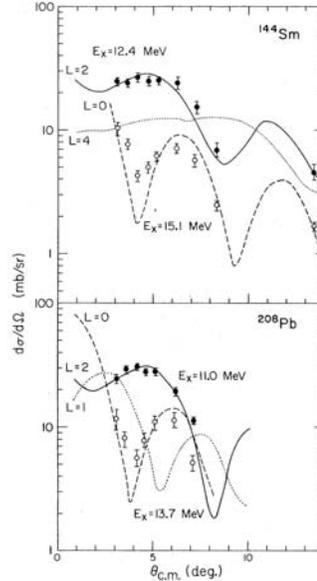


FIG. 2. Angular distributions obtained for both components of the giant resonance peaks in  $^{144}\text{Sm}$  and  $^{208}\text{Pb}$ . DWBA calculations are shown for several  $L$  transfers. The normalizations for the  $L=1$  and  $L=4$  calculations are arbitrary.

$^{208}\text{Pb}$  by Harakeh *et al.*<sup>6</sup> The angular distribution obtained for the two broad components for both  $^{144}\text{Sm}$  and  $^{208}\text{Pb}$  are shown in Fig. 2. In each case the angular distributions for the two GR components are different as is apparent from the spectra shown in Fig. 1.

Distorted-wave Born-approximation (DWBA) calculations were performed using the computer code DWUCK.<sup>8</sup> The calculations used were performed with the parameters listed in Ref. 1

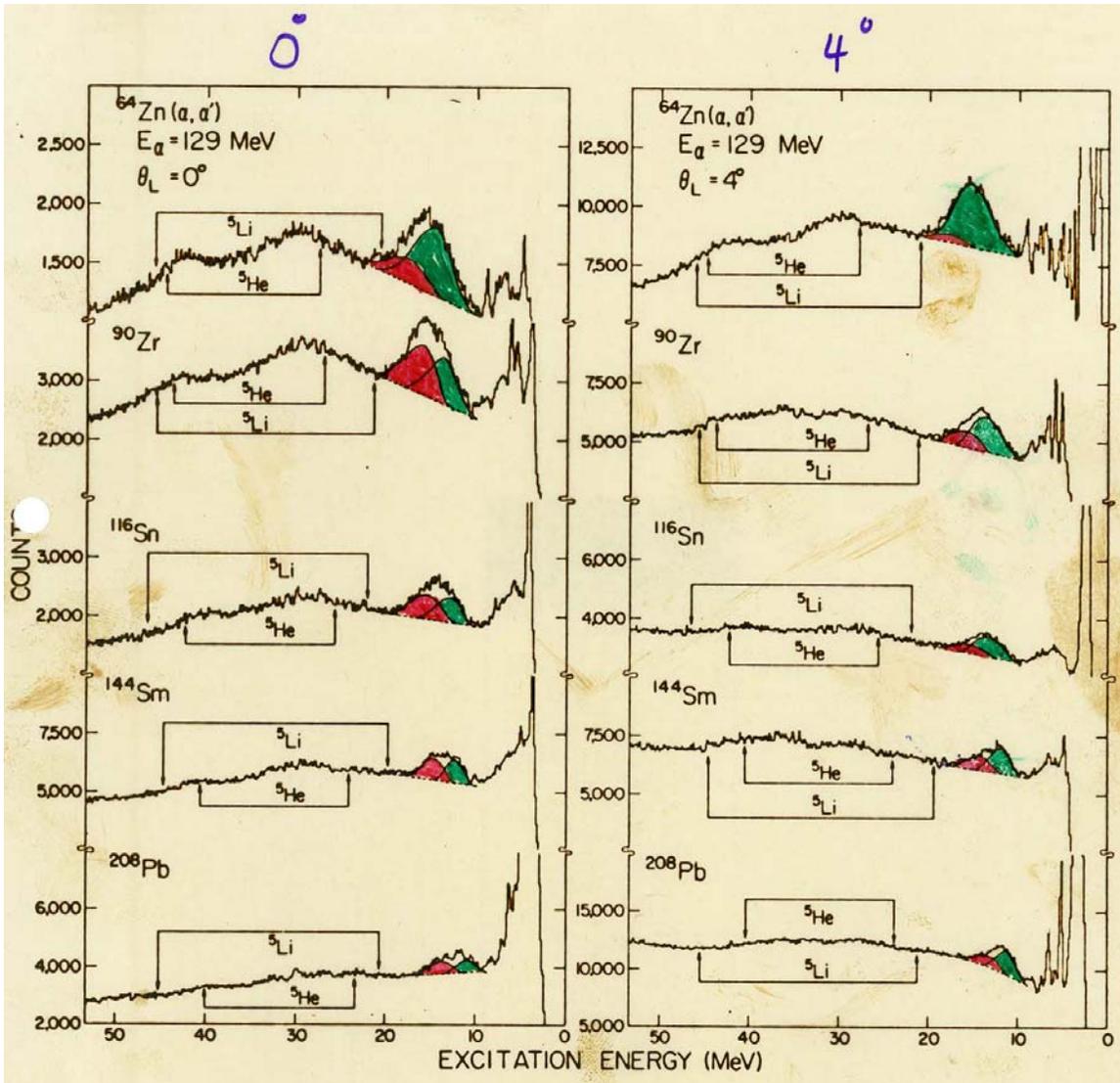
TABLE I. Parameters obtained for the two components of the GR peak.

	$E_x$ (MeV)	$\Gamma$ (MeV)	$J^\pi$	$\beta^2 R^2$	EWSR (%)
$^{144}\text{Sm}$	$12.4 \pm 0.4$	$2.6 \pm 0.4$	$2^+$	0.43	$85 \pm 15$
	$15.1 \pm 0.5$	$2.9 \pm 0.5$	$0^+$	0.22	$100 \pm 20$
$^{208}\text{Pb}$	$11.0 \pm 0.2$	$2.7 \pm 0.3$	$2^+$	0.35	$90 \pm 20$
	$13.7 \pm 0.4$	$3.0 \pm 0.5$	$0^+$	0.17	$105 \pm 20$

( $^{148}\text{Sm}$  parameters were used for  $^{144}\text{Sm}$ ). Several optical-potential-parameter sets from the literature were tried, but the results were roughly independent of optical parameters. Monopole calculations were performed using both Satchler's<sup>9</sup> version-1 and -2 form factors. The other form factors and sum rules used are discussed in Ref. 1. The magnitudes of the DWBA predictions changed somewhat with differing optical potentials, differing form factors (for the monopole), and differing Coulomb-excitation parameters; however, the shapes of the angular distributions were essentially unchanged. The predictions for a monopole state, the isovector-dipole state, a quadrupole state, and a hexadecapole state are shown superimposed on the data in Fig. 2. It is readily seen that the lower-excitation component is relatively well fitted by the quadrupole calculation, while the higher-excitation component is fitted adequately by the monopole calculation. In particular, the predicted signature of a monopole state, a sharp minimum around  $4^\circ$ , is very appar-

later we succeeded at  $0^\circ$ .

BUT average angle  $\sim 2^\circ$ .



## A Few Facts About the GMR

$$E_{\gamma} = \hbar \omega \sim 15 \text{ MeV} \Rightarrow f \sim 4 \times 10^{21} / \text{s}$$

$$\tau \sim \hbar / \Gamma \sim 3 \text{ MeV} \Rightarrow \tau \sim 6 \times 10^{-22} \text{ s}$$

Thus it "lives" for at most a few oscillations!

$$\text{Oscillation Amplitude: } \frac{\delta \rho}{\rho} \sim 0.05$$

Interesting Numbers from  $K_{nm} \approx 230 \text{ MeV}$

$$\text{Compressibility } \chi_{nm} = \frac{9}{\eta K_{nm}} \Rightarrow \chi_{nm} \sim 1.5 \times 10^{-32} \left[ \frac{\eta}{\text{m}^2} \right]^{-1}$$

$$\eta = \text{nucleon density} \quad \chi_{\text{water}} \sim 5 \times 10^{-10} \left[ \frac{\eta}{\text{m}^2} \right]^{-1}$$

Velocity of Sound in nuclear matter

$$\begin{aligned} c_s &= \sqrt{B/\rho} \\ &= \sqrt{\frac{1}{\chi_{nm}} m \eta} \\ &= \left[ \frac{K_{nm}}{9 m c^2} \right]^{1/2} c \end{aligned}$$

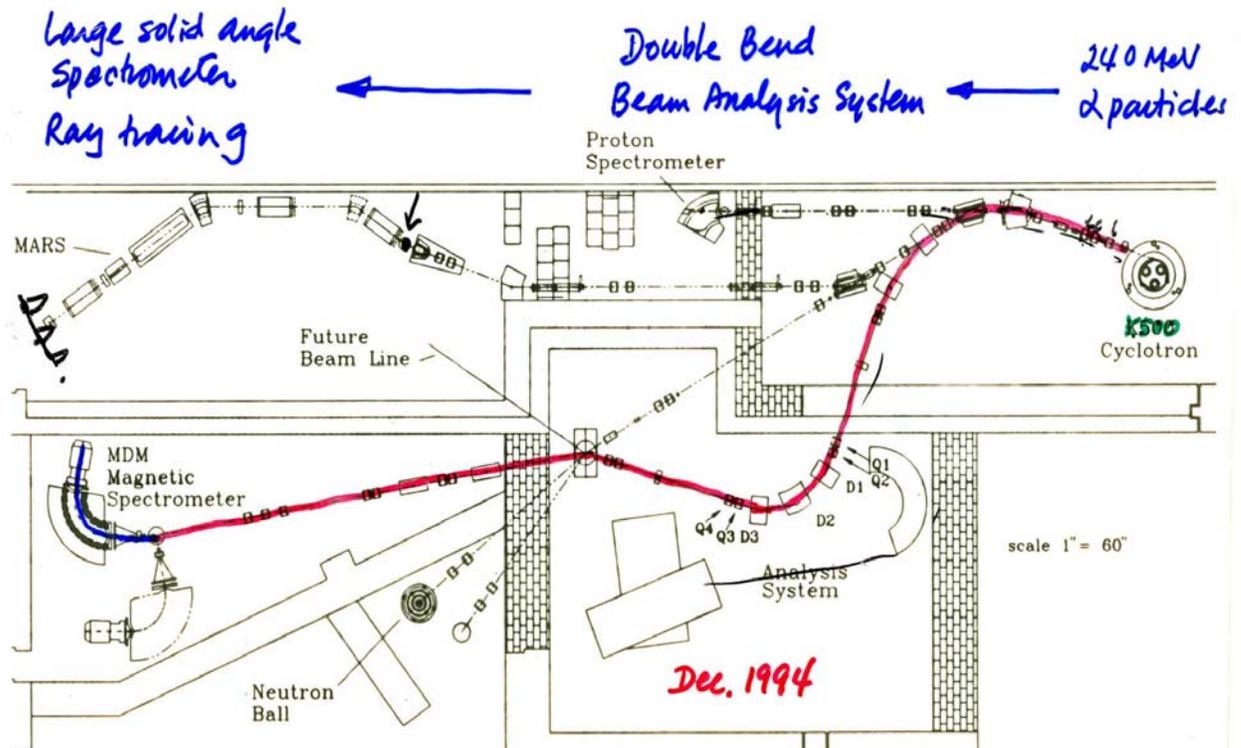
$$c_s = 0.15 c$$

**Built:**

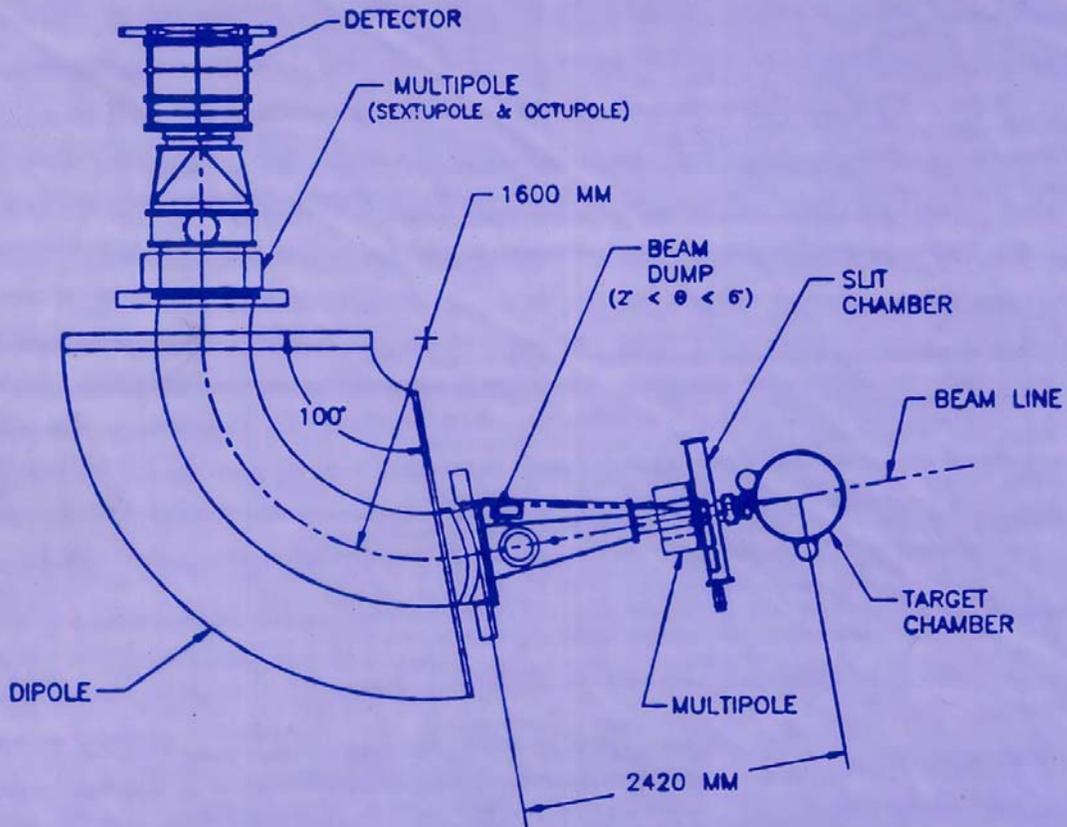
**New cyclotron - higher energy.**

**New beam analysis/transport system - Clean beams.**

MDM spectrometer

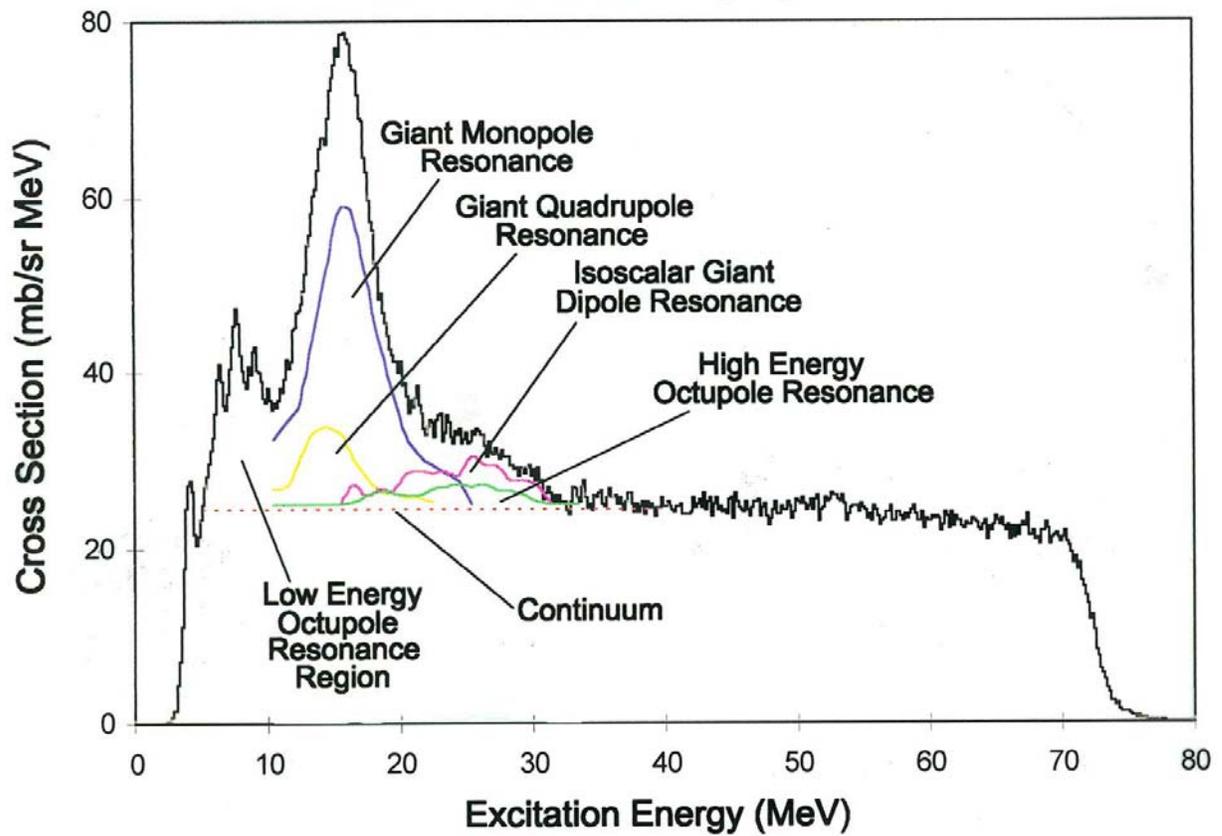


# MDM Spectrometer



K	315–400 (315 alphas, 78 MeV/A)
Vertical Acceptance	+50 mrad
Horizontal Acceptance	+40 mrad
Solid Angle	8 msr
$E_{\max}/E_{\min}$	1.31 ( $E_x$ up to 50 MeV @ 200 alphas)
Resolving Power $\Delta E/E$	4500

240 MeV Alpha Particles +  $^{90}\text{Zr}$   
Center-of-Mass Scattering Angle =  $1.08^\circ$

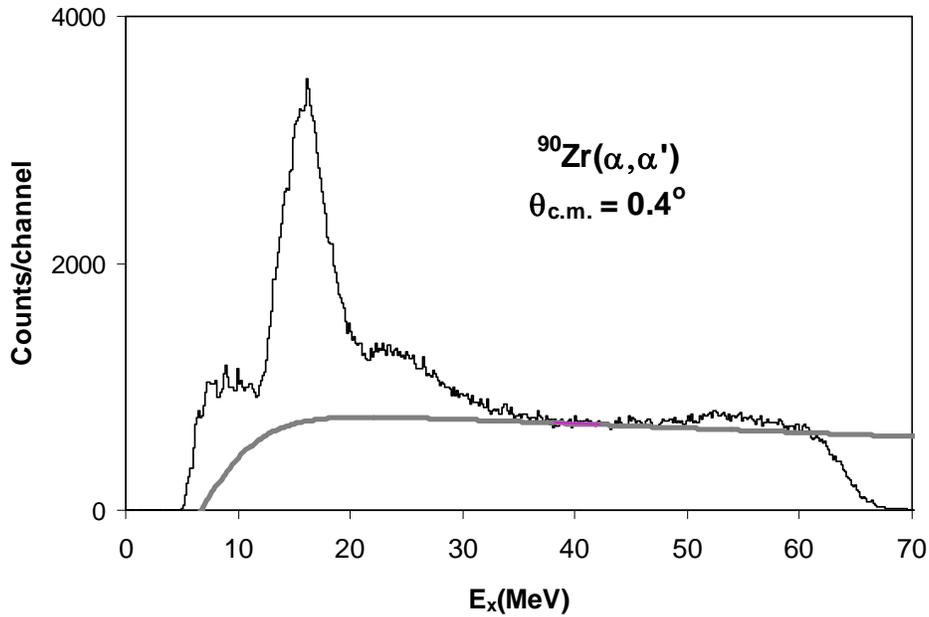


**Measuring only horizontal angle.**

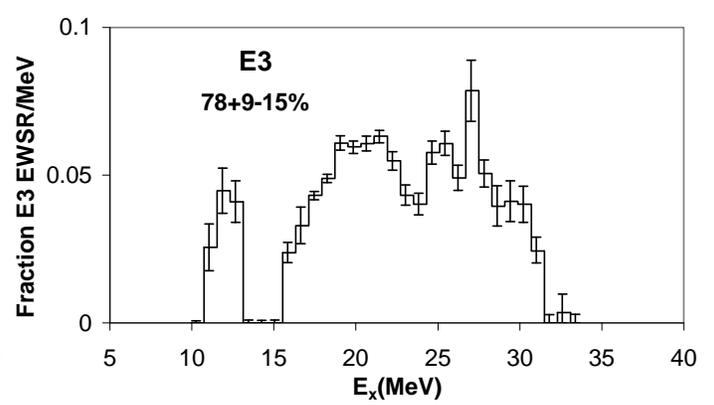
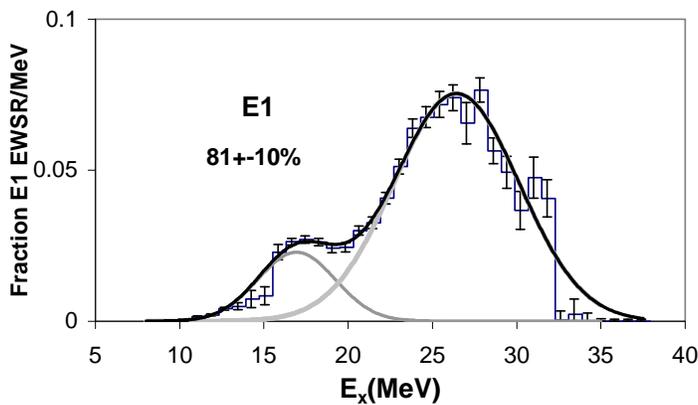
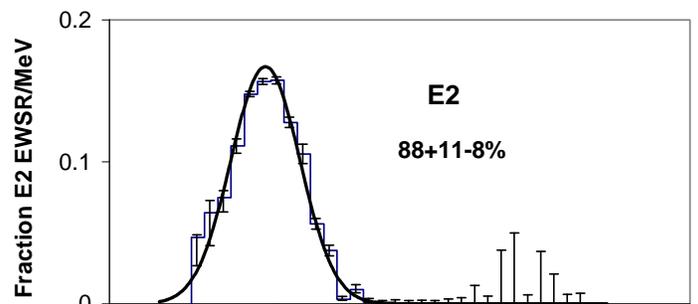
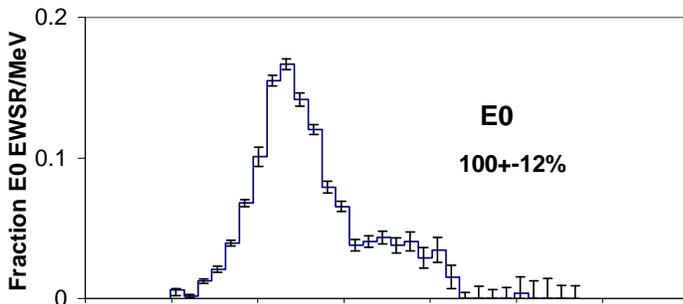
**Average over vertical angle.**

# Added measurement of vertical angle

## $^{90}\text{Zr}$ Spectrum



## Multipole Distributions



**We can separate multipoles!**

## An Experimentalist's View of Nuclear Compressibility

Important Quantities which characterize Nuclear Matter ARE:

(1) Binding Energy per particle:  $\sim 16 \text{ MeV/particle}$   
obtained from a semi-empirical mass formula  
with fits to nuclear masses.

(2) Density  $\rho_0 \sim 0.17 \text{ nucleons/fm}^3$  - or -  
Fermi momentum  $k_F \sim 1.36 \text{ fm}^{-1}$   
obtained from central density of heavy  
nuclei measured by electron scattering

(3) Compression Modulus (Incompressibility)

$$K \equiv \left( k_F^2 \frac{d^2 E/A}{dk_F^2} \right)_0$$

obtained from energy of the breathing  
mode state.

$$E_{\text{BM}} = \frac{\hbar}{r_0} \sqrt{\frac{K_A}{m}}$$

$$K_A = r_0^2 \frac{d^2 E/A}{dr_0^2} \quad [\text{for a finite nucleus}]$$

$$E_{\text{GMR}} = \hbar (K_A / m \langle r^2 \rangle)^{1/2}$$

$\langle r^2 \rangle$ : mean square nuclear radius

m: nucleon mass

$K_A$ : compressibility of nucleus

## Compressibility of Nuclear Matter

### Simple Picture: Leptodermous Expansion

$$K_A = K_{\text{NM}} + K_{\text{Surf}} A^{-1/3} + K_{\text{vs}} \left( (N-Z) / A \right)^2 + K_{\text{Coul}} * Z^2 / A^{4/3}$$

$$K_{\text{vs}} = K_{\text{Sym}} + L (K' / K_v - 6)$$

Where  $K_{\text{NM}}$ : curvature of  $E/A$  around  $\rho_0$

$K_{\text{Sym}}$ : curvature of symmetry energy

Note that  $E_{\text{GMR}}$  depends on  $K_{\text{NM}}$  AND  $K_{\text{Sym}}$ !

### The Right Way to get $K_{\text{NM}}$

Calculate  $E_{\text{GMR}}$  using effective interactions  
(each results in a specific  $K_{\text{NM}}$ )

Compare to experiment!

Vretenar et al. PRC 68, 024310 (2003).

Agrawal et al. PRC 68, 031304 (2003).

Colò et al. PRC 70, 024307 (2004).

Role of  $K_v$  and  $K_{\text{Sym}}$  in Infinite nuclear matter.

## Microscopic Calculations:

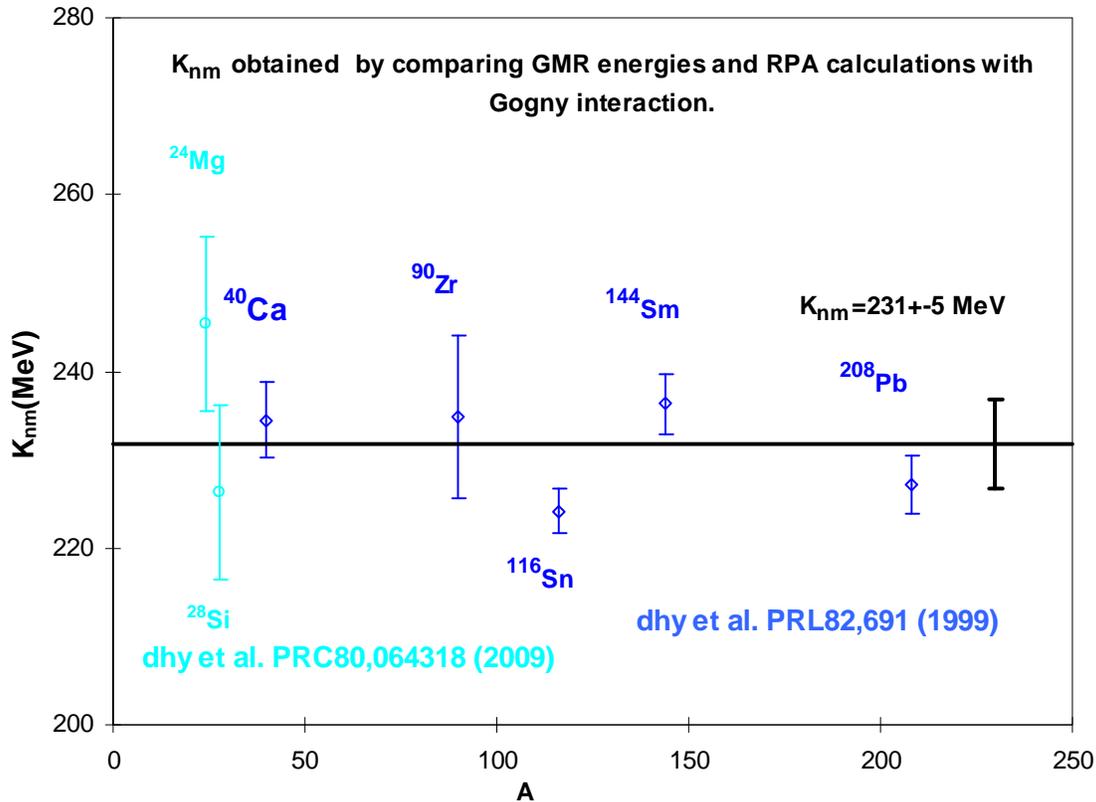
Non\_Relativistic:

Skyrme, Gogny effective interactions.

Relativistic:

NL1, NL3, etc. parameter sets.

Compare calculated to experimental  $E_{\text{GMR}}$



$^{24}\text{Mg}$ ,  $^{28}\text{Si}$  Péru, Goutte, Berger, NPA788, 44 (2007)  
QuasiParticle RPA-HFB Gogny D1S

$K_{\text{NM}}$  interaction dependent.

Symmetry Energy  $\rho$  dependence.

Present:  $K_{\text{NM}} \sim 220-240$  MeV

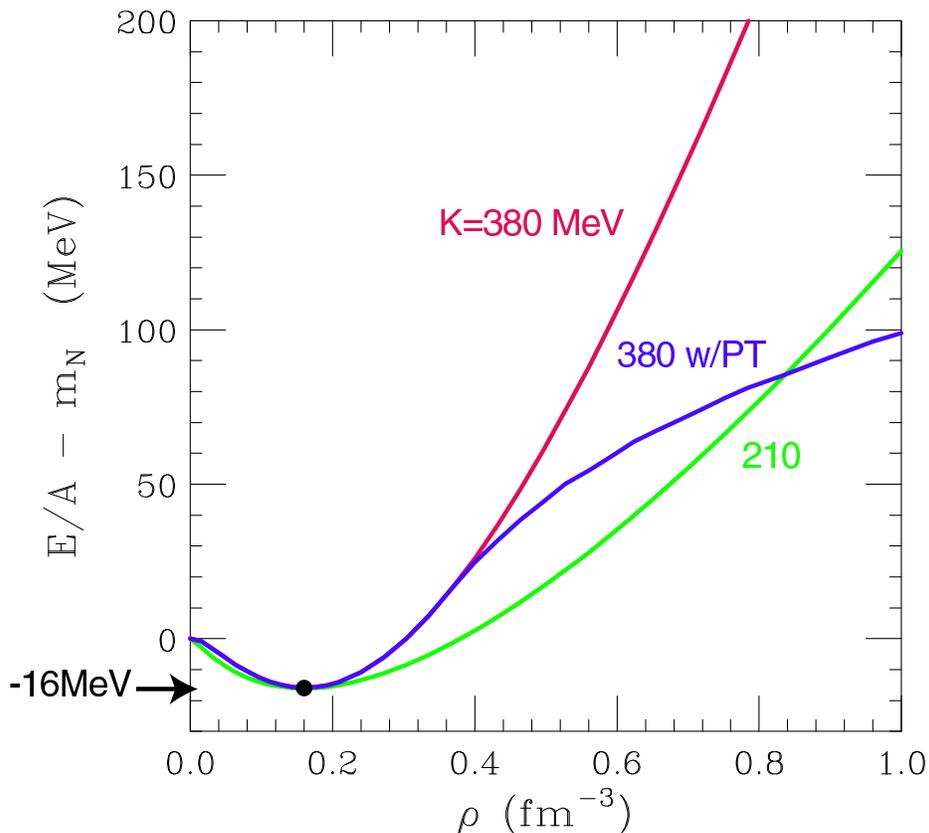
# Nuclear Matter Equation of State

A parametrization of the EOS to order  $\varepsilon^2$ :  
(M. Farine et al. NPA 615,135 (1997))

$$E/A = A_v + (K_{NM}/18) \varepsilon^2 + [(\rho_n - \rho_p) / \rho]^2 \{J + (L/3) \varepsilon + (K_{SYM}/18) \varepsilon^2 + \dots\} + \dots$$

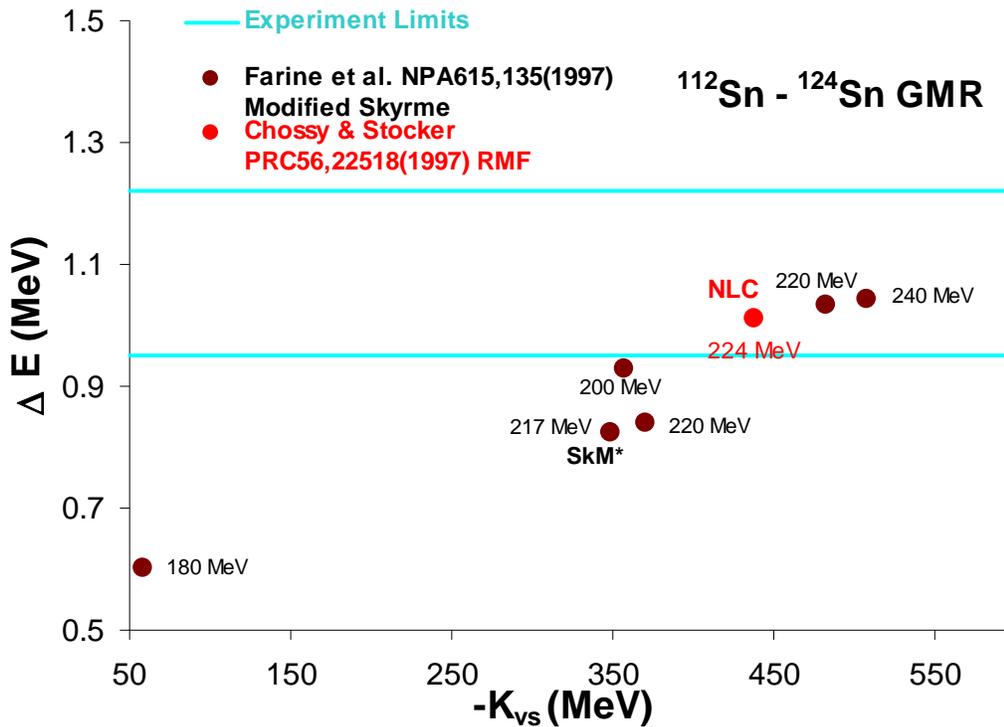
Where  $\varepsilon = (\rho - \rho_0) / \rho_0$  and  $(\rho_n - \rho_p) / \rho \sim (N - Z) / A$

Second derivative leaves  $K_{NM}$  and  $K_{SYM}$ .

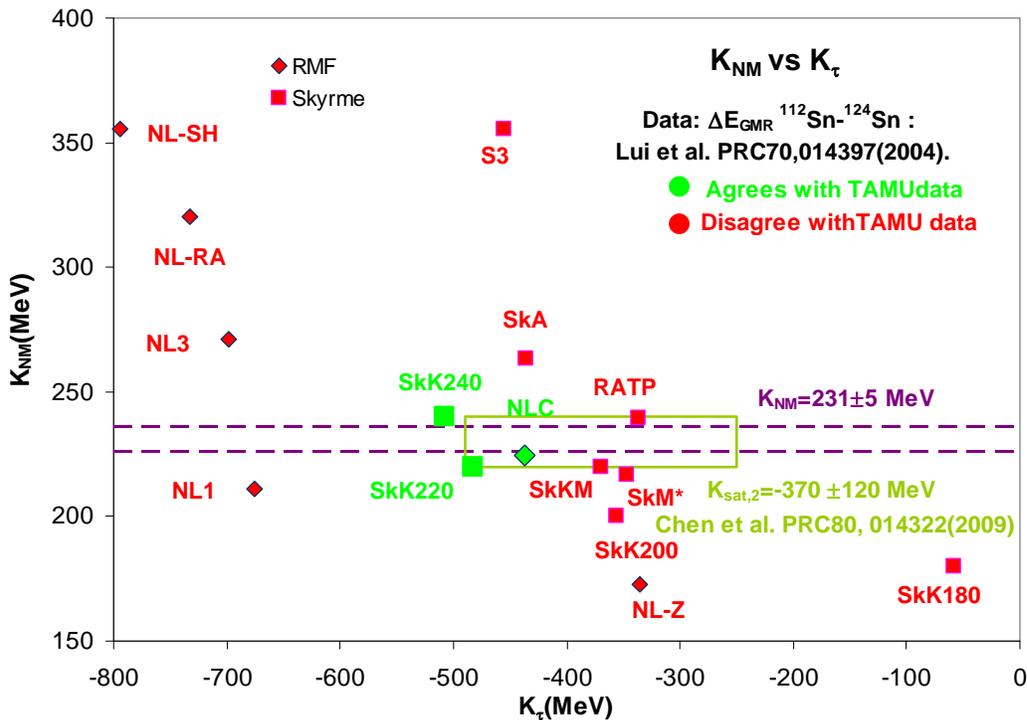


$K_{SYM}$  goes as  $(N-Z)/A)^2$

**To get  $K_{SYM}$ : Change  $(N-Z)/A$   $^{112-124}\text{Sn}$**



$$K_{vs} = K_{\tau} = K_{SYM} + L(K' / K_v - 6) \quad K_{\tau} < -375 \text{ MeV}$$



**Increase (N-Z/A) for  $K_{\text{sym}}$**

**Move away from stable nuclei**

### **Present Research**

**D.H. Youngblood**

**Y.-W.Lui**

**Jonathon Button(thesis project)**

**Will McGrew (REU)**

**Yi Xu (post doc joins 8/30/12)**

**Hanyu Li (undergrad joins 9/1/12)**

**Use unstable nuclei as beams:**

**upgraded Cyclotron facility.**

**Inverse reactions – Problem: Helium target**

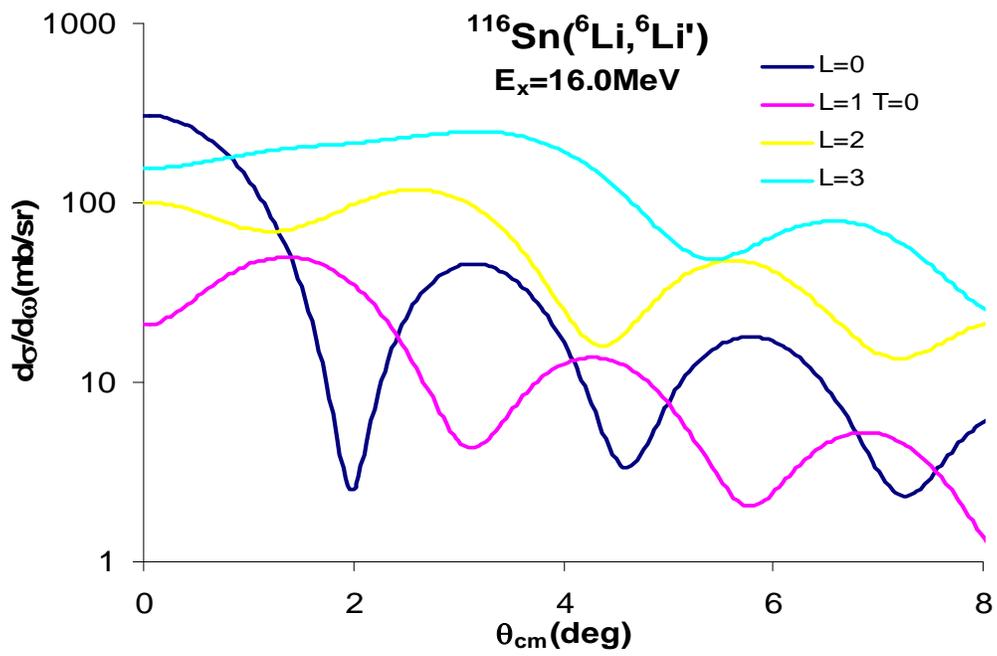
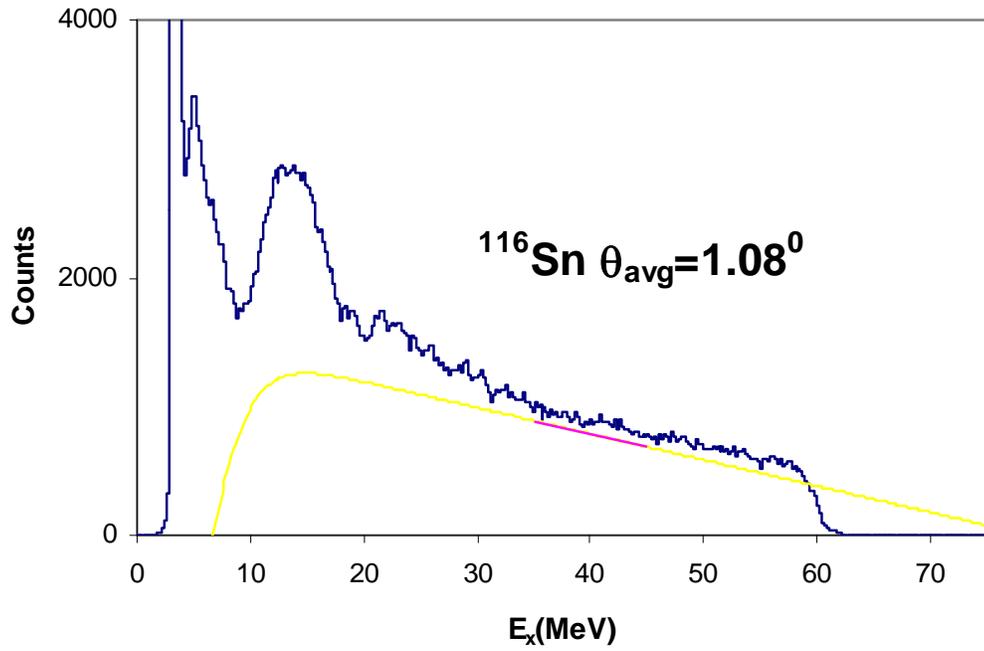
**Solution: Use  ${}^6\text{Li}$  target**

**X. Chen's Thesis: 240 MeV  ${}^6\text{Li}$  on  ${}^{24}\text{Mg}$ ,  ${}^{28}\text{Si}$ ,  ${}^{116}\text{Sn}$ .**

**Prove  ${}^6\text{Li}$  scattering good for GMR.**

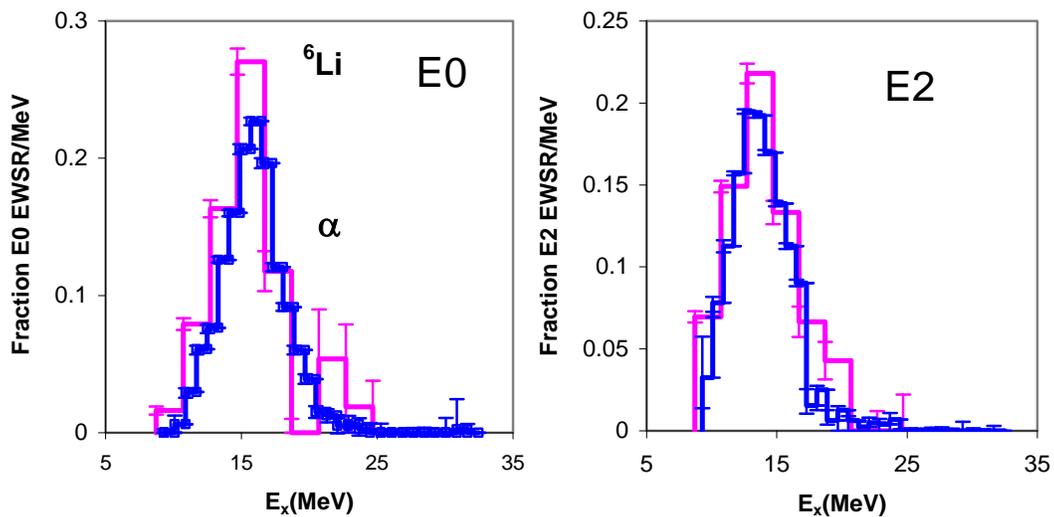
# Inelastic Scattering to Giant Resonances

240 MeV  ${}^6\text{Li}$  +  ${}^{116}\text{Sn}$

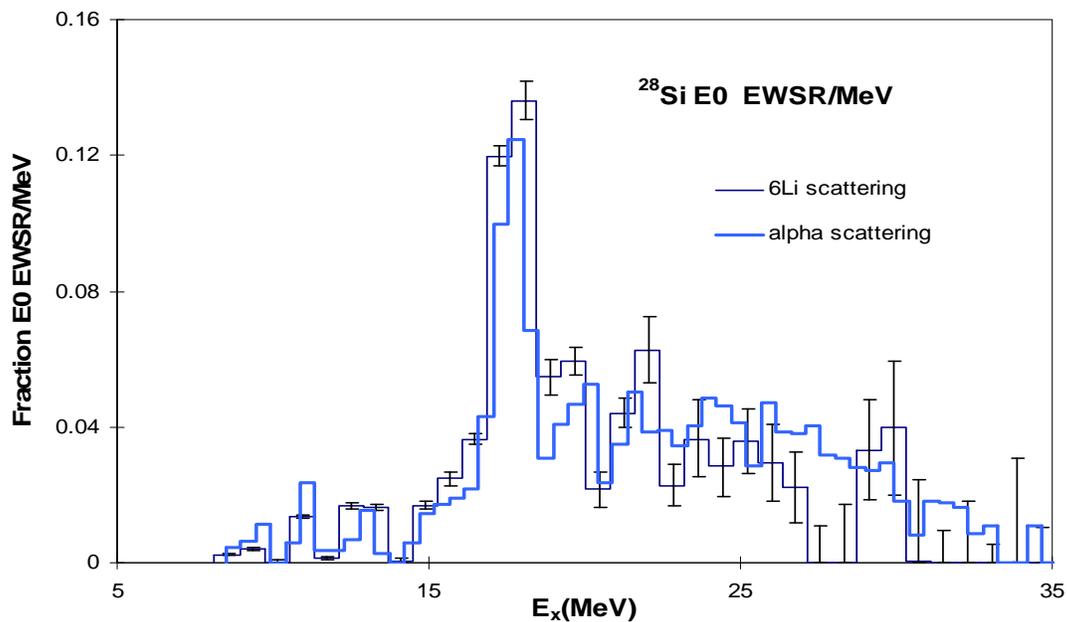


# Multipole Distributions

$^{116}\text{Sn}$

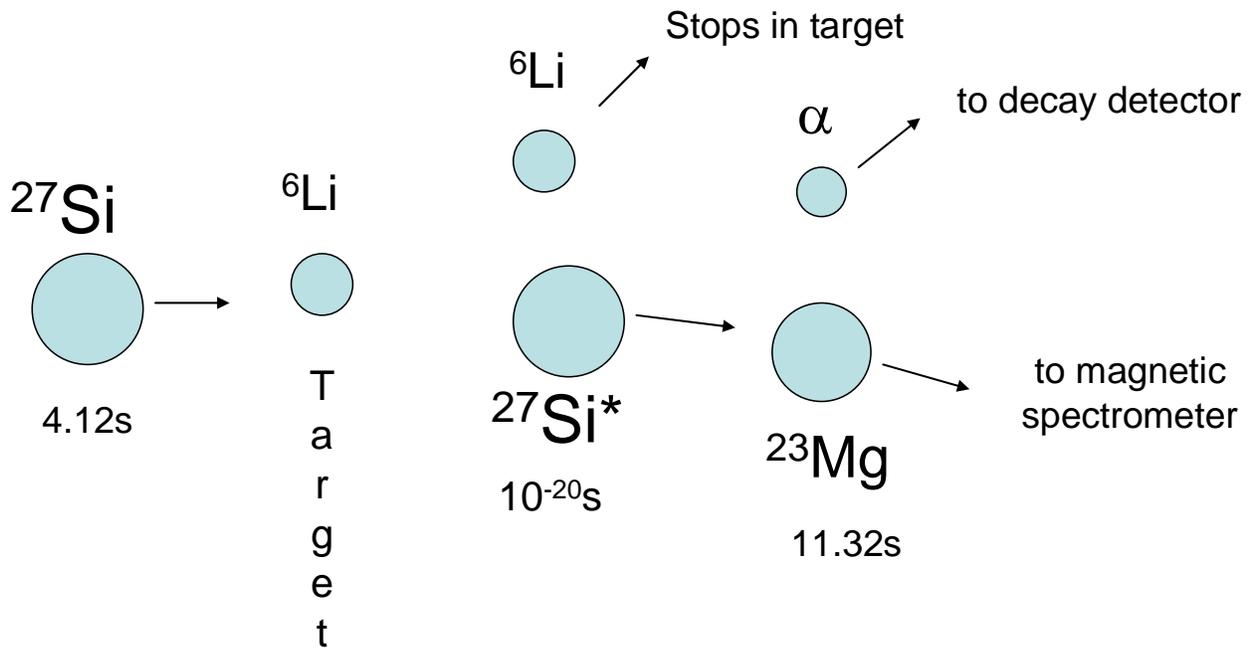


$^{28}\text{Si}$  GMR

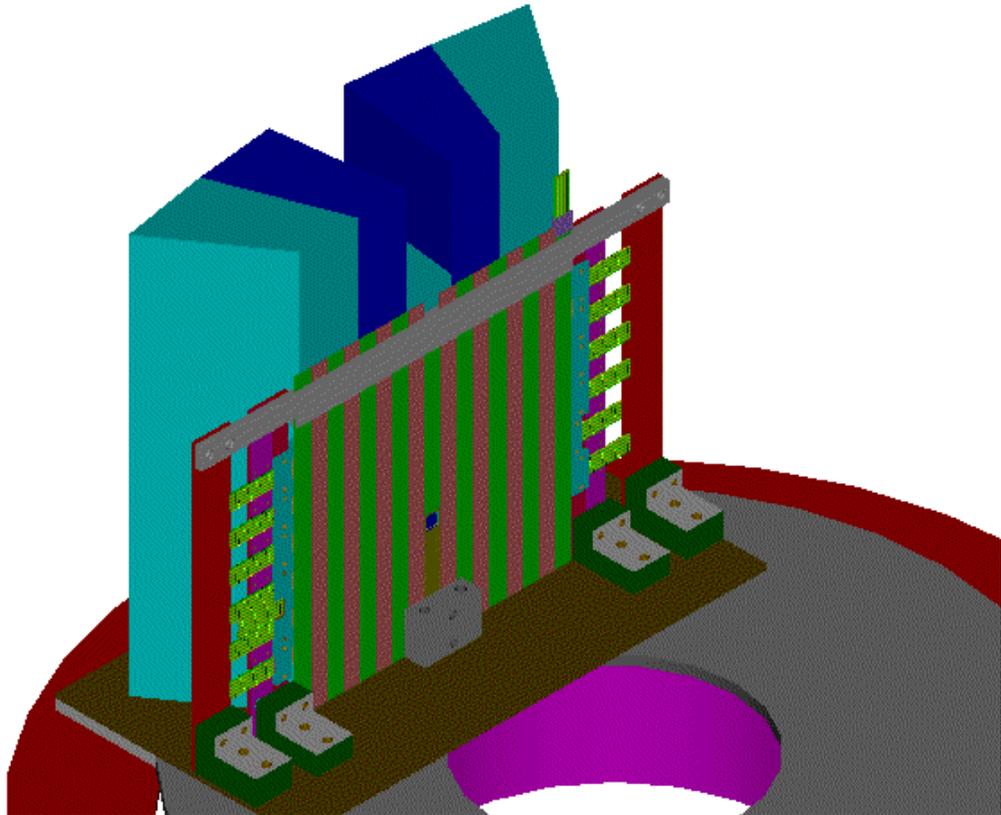


$^6\text{Li}$ ,  $\alpha$  agree for GR's  $^{116}\text{Sn}$ ,  $^{28}\text{Si}$ ,  $^{24}\text{Mg}$ .

# To study GMR in $^{27}\text{Si}$



## **Decay detector – Jonathan Button thesis**



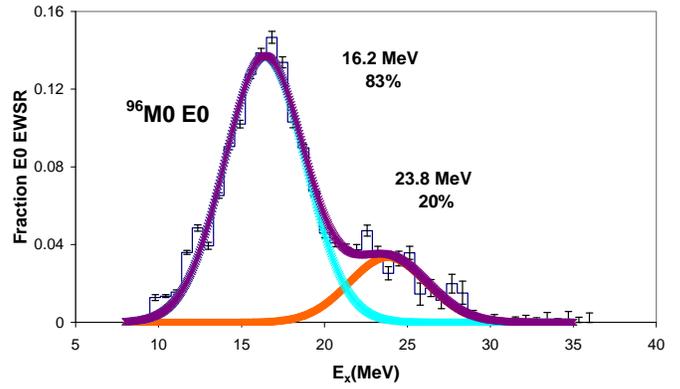
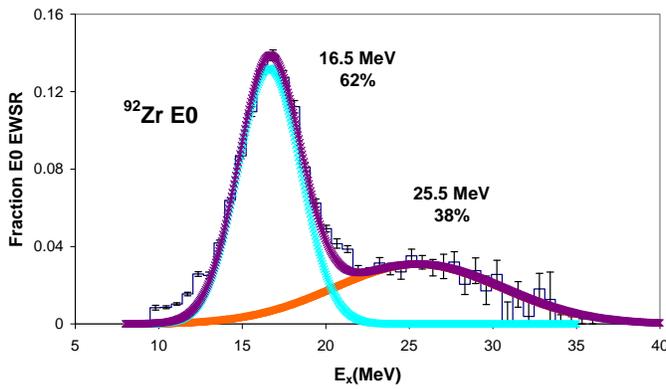
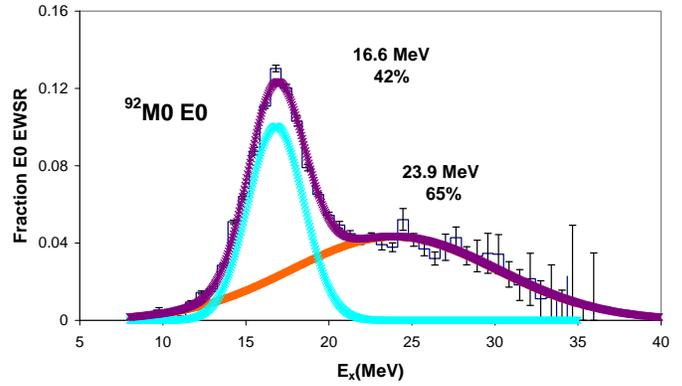
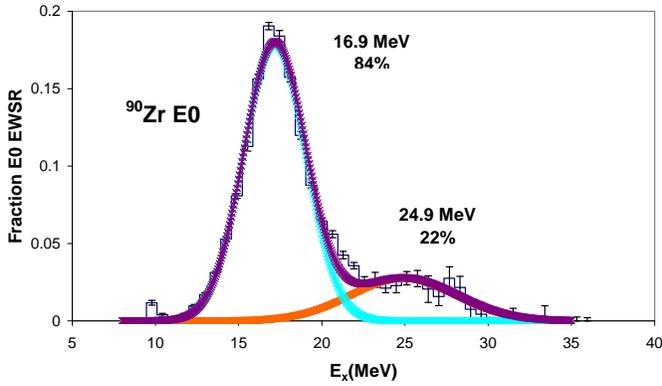
**Will McGrew: Analyzing data**

**Test run**

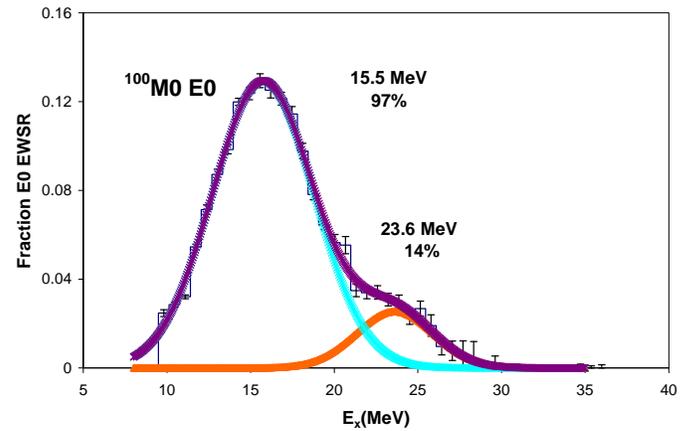
**Calibration**

**Designing Faraday Cup/Beam Stop**

# Other Monopole Things

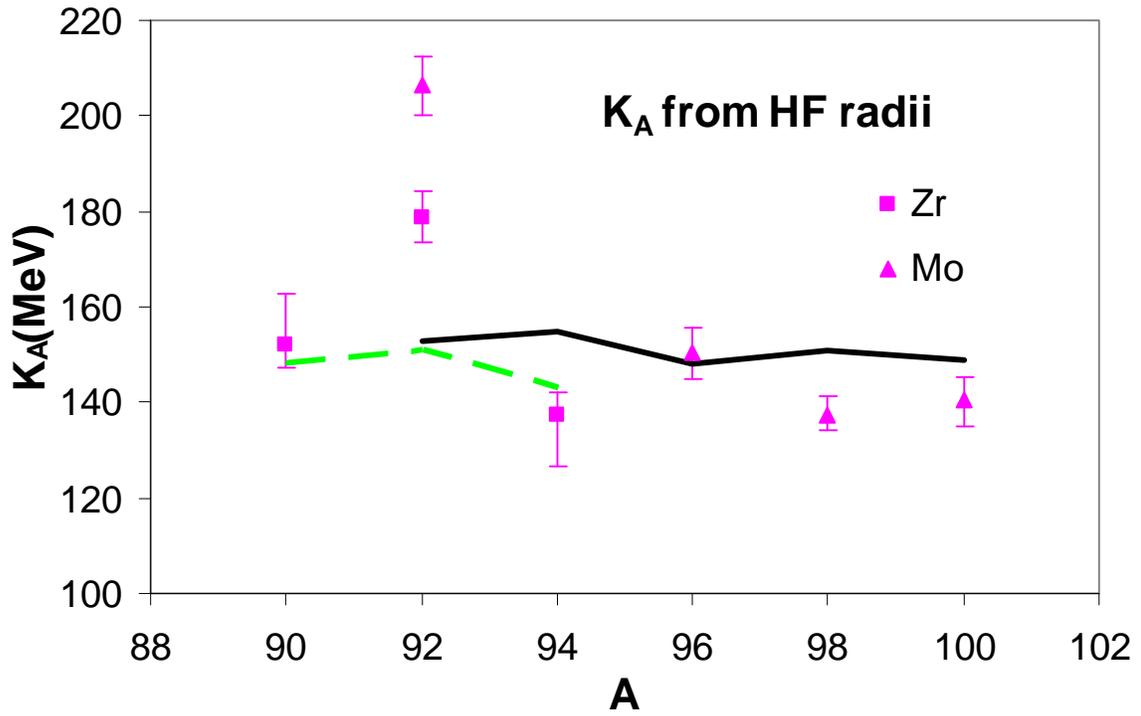


m1/m0	
<sup>90</sup> Zr	17.87 MeV
<sup>92</sup> Mo	19.62 MeV



$E_{GMR}^{92Mo}$  should be below  $^{90}Zr$ ??

**$K_A$  for Mass 92 very different.**



**$^{92}\text{Mo}$**

**$K_A$   $5\sigma$  from expected value!**

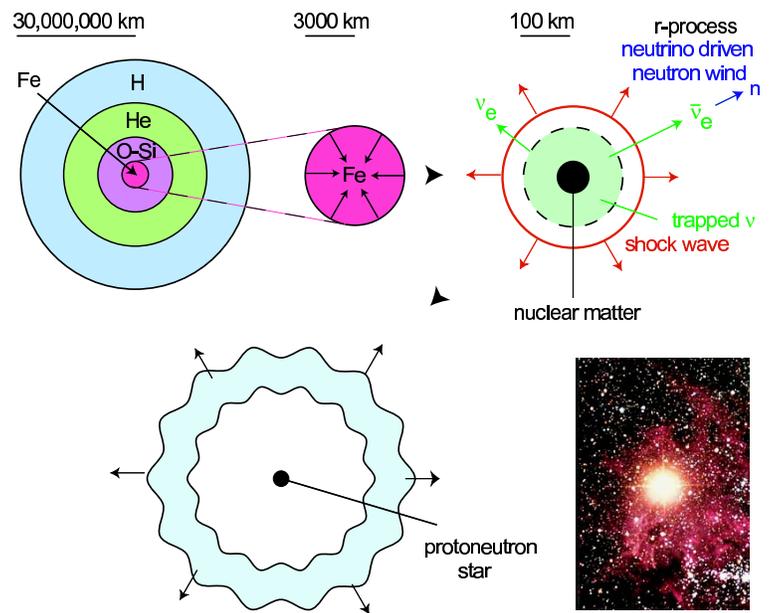
Nuclear equation of state influences many **astrophysical processes**

**Double pulsar rotation** (astro-ph 0506566)

**Binary mergers** (astro-ph 0512126)

**Neutron star formation:**

Life of a type II supernova



Protonneutron star has about the same density as nuclei

**Stolen from :**

**C. Hartnack and J. Aichelin**

**Subatech/University of Nantes**

**H. Oeschler**

**Technical University of Darmstadt**

## Sum Rules

Electromagnetic operator  $Q_L = \sum_i r_i^L Y_{LM}(\mathbf{e}_i)$

Show  $\sum_n E_n |\langle n | Q_L | 0 \rangle|^2 = \langle 0 | [[Q_L, H], Q_L] | 0 \rangle = S_L$

Assumes  $[Q_L, V] = 0$

Then evaluate  $\langle 0 | [[Q_L, H], Q_L] | 0 \rangle$  using commutator relations

Result  $S_L = \sum_n E_n B(E_L \uparrow) = \frac{\hbar^2(L)(2L+1)A}{8\pi m} \langle r^{2L-2} \rangle$  (for  $L \geq 2$ )

\* A "giant resonance" is a state carrying a significant fraction of this sum.

For Inelastic  $\alpha$  scattering

$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}} = \beta^2 \left(\frac{d\sigma}{d\Omega}\right)_{\text{DWBA}}$   $\beta$  - deformation parameter

$B(E2 \uparrow) = \beta^2 R^4 \left(\frac{3Z}{4\pi}\right)^2$

$\beta = R_2 = \beta_m R_m$

$R^2$  evaluated for a Fermi distribution

Definition of  $\beta$

$R(\theta, \phi) = R \left[ 1 + \sum_{LM} \alpha_{LM} Y_{LM}(\theta, \phi) \right]$

$\beta_L(n) = (2L+1)^{1/2} \langle n, M | \alpha_{LM}^* | 0, 0 \rangle$

Deformed optical potential used for form factor in DWBA.